Radiation Pressure Dominated Plasma Flow

P. Mulser and G. Spindler

Projektgruppe für Laserforschung der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V., D-8046 Garching

Z. Naturforsch. **34a**, 1059–1062 (1979); received July 6, 1979

The influence of light pressure on plane and spherical plasma flows is treated. Numerical calculations show that smooth laser pulses lead to formation of an overdense plateau if thermal transport is non-negligible there, and that an originally supersonic flow becomes subsonic in this region. The results are consistent with analytical considerations. Density humps, as reported in the literature, are found in conjunction with spiking pulses or prepulses only.

In laser plasma interaction the radiation pressure appears as a force additional to the hydrodynamic pressure and leads to a variety of modified density and flow velocity profiles. They can be divided into two classes: transient structures which are induced by very short pulses or at the very beginning of a fast rising pulse and quasi-steady-state density profiles produced by longer, smooth pulses. Transient structures have been theoretically investigated by many authors. In Refs. [1–3] their soliton character is treated, in Refs. [4] and [5] the resonance properties of density modifications are stressed. They all have in common that no essential flow of the plasma has been taken into account. On the other hand it is well known that the flow of the plasma plays a decisive role [6–11]. In laser plasmas produced from dense targets for instance the flow is subsonic near the target and becomes supersonic with decreasing density.

A realistic treatment of density structures has to be based on hydrodynamics (at least) and the light wave equation. This was first done in Ref. [8] for a plane subsonic plasma flow behind the critical point. If the flow is supersonic and divergent there, moderate laser intensities may lead to the formation of plateaus just at the critical density [9]. For higher intensities it has been suggested [10] that a stationary shock wave forms behind the critical point and the flow changes from supersonic to subsonic. The existence of such density humps under certain conditions has been shown numerically in Refs. [7] and [11].

Reprint requests to Frau Dr. L. Johannsen, Max-Planck-Institut für Plasmaphysik, Projektgruppe für Laserforschung, D-8046 Garching/München.

0340-4811 / 79 / 0900-1059 \$ 01.00/0

In this letter we show that concise — and at the same time exact — formulae for the variations of density and Mach number in steady adiabatic and isothermal flows can be deduced if all quantities are evaluated at any local maximum of the electric field amplitude. — We then solve the correct initial-value problem of plasma production from a solid target under the action of light pressure. We find the phenomenon of overdense plateau formation not yet reported in literature.

A) Analytical considerations. Let us consider a slightly divergent stationary plasma flow so that it can be approximated as plane over a few local light wavelengths. Owing to the interaction of the light pressure with the plasma the flow is divided into two regions: the overdense region 1, in which the light wave becomes evanescent, and the underdense region 2, in which the flow becomes definitively supersonic as the distance from the absorbing region increases. The width of the transition zone at the critical density ϱ_c is smaller than $\lambda/2$ and decreases with increasing laser intensity. The force density exerted by the radiation pressure is defined as the time averaged Lorentz force over one oscillation period, i.e.

$$\varkappa = -e\,n_e \langle \vec{E} + \vec{v} \times \vec{B} \rangle \tag{1}$$

$$= -\frac{e^2 n_e}{4 m_e(\omega^2+\nu^2)} \Big\{ \nabla(\hat{E}\hat{E}^*) + 2\frac{\nu}{\omega} \nabla (\mathrm{Re}\,\hat{E}\cdot \mathrm{Im}\,\hat{E}_c - \mathrm{Re}\,\hat{E}_c \cdot \mathrm{Im}\,\hat{E}) \Big\},$$

where $\hat{E}$ is given by $\vec{E} = \hat{E}(x,t)\,e^{-i\omega t}$ and ν is the collision frequency (this definition of $\varkappa$ coincides with that given in Ref. [12]. The subscript c indicates that in taking the gradient this component has to be treated as a constant. Because of

$\frac{\partial}{\partial x}\varrho v = 0$ the stationary momentum equation reads $\frac{\partial}{\partial x}(\varrho v^2 + p) = \varkappa$. For normal incidence of radiation $\varkappa$ can be integrated from $x_1 < x_c$ to $x_2 > x_c$ by means of the steady-state wave equation

$$\hat{E}'' + k^2\left(1 - \frac{\varrho}{\varrho_c(1 + i\,\nu/\omega)}\right)\hat{E} = 0$$

and yields

$$-\int_{x_1}^{x_2} \varkappa\, dx = \beta\,\varrho_c[\hat{E}_2\hat{E}_2^* - \hat{E}_1\hat{E}_1^* + k^{-2}(|\hat{E}_2'|^2 - |\hat{E}_1'|^2)],$$
$$\beta = \frac{e^2}{4 m_e m_i \omega^2}.$$

If we take x_1 sufficiently far in the evanescent region ($E_1 \simeq 0$) and x_2 at the maximum of $\hat{E}\hat{E}^*$, the momentum balance assumes the simple form

$$\varrho_1 v_1^2 + p_1 = \varrho_2 v_2^2 + p_2 + \beta\,\varrho_c\,|\hat{E}|^2_{\max}, \quad (2)$$

since owing to $\nu^2/\omega^2 \ll 1$ $|\hat{E}'|$ can be replaced by $|\hat{E}|'$. Assuming now a polytropic equation of state, $p\,\varrho^{-\gamma} = \text{const}$ (this is compatible since $\varkappa$ is a function of position only), introducing the Mach number $M = v/s$, $s^2 = \gamma p/\varrho$, and eliminating ϱ in the momentum equation, we obtain

$$\left(M - \frac{1}{M}\right)\frac{\partial M}{\partial x} = -\frac{\gamma+1}{2}\frac{\beta}{s^2}\frac{\partial}{\partial x}|\hat{E}|^2 \quad (3)$$

for $\nu/\omega \ll 1$. From this relation it is immediately deduced that in region 1 the flow has to be subsonic with respect to x_c ($M < 1$), and that transition to supersonic flow ($M > 1$) occurs in a maximum of $\hat{E}\hat{E}^*$, at the latest in the highest maximum, which is usually that nearest x_c. Integration of Eq. (3) from x_1 to x_2 yields the Mach number as a function of the maximum light pressure $p_L = \beta\,\varrho_c|\hat{E}|^2_{\max} = \frac{\varepsilon_0}{4}|\hat{E}|^2_{\max}$:

$$M_1^2 + \frac{2}{\gamma-1} - \frac{\gamma+1}{\gamma-1} M_1^{2(\gamma-1)/(\gamma+1)} = \frac{2 p_L}{s_1^2\,\varrho_c}. \quad (4)$$

From Eqs. (2) and (4) it then follows that

$$\frac{\varrho_1}{\varrho_c} = \left(\frac{p_L}{p_c}\right)^{1/\gamma}(1 + \gamma M_1^2 - (\gamma+1)\,M_1^{2\gamma/(\gamma+1)})^{-1/\gamma},$$
$$\varrho_2 = \varrho_1 M_1^{2/(\gamma+1)}, \quad p_c = \varrho_c s_c^2/\gamma, \quad (5)$$

i.e. light pressure leads to a density step at the critical density. Of particular interest is the case of an isothermal plasma ($\gamma \to 1$), for which expressions (4) and (5) yield the following relations:

$$M_1^2 - \ln M_1^2 - 1 = 2\frac{p_L}{p_c},$$
$$\frac{\varrho_1}{\varrho_c} = \frac{p_L}{p_c(1 - M_1)^2}, \quad \varrho_2 = \varrho_1 M_1. \quad (6)$$

B) Numerical treatment. Laser-produced plasmas are divergent. The simplest way to study such flows is to assume spherical symmetry. We consider a sphere of radius r_0 and density ϱ_0 on the surface of which laser radiation of a given intensity φ_0 impinges normally. For simplicity, we assume that the fraction of incident light which creates and heats the plasma is absorbed at the critical density. We assume further that the plasma is in the charge state $Z = 1$, and that only the electrons are heated. Heat conduction is included in a classical form with the heat flux density q: $q = -\varkappa_0 T^{5/2}\partial T/\partial r$, with the upper limit $q \leqq q_{\max} = f n_e (k_B T)^{3/2}/m_e^{1/2}$, where f is a number not larger than unity. We introduce the normalized density R: $\varrho = \varrho_c R$; the governing equations are then (ideal gas, $p = \varrho k_B T/m_i$)

$$\frac{\partial R}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 R v) = 0, \quad (7)$$

$$\frac{dv}{dt} = -\frac{k_B}{m_i}\frac{1}{R}\frac{\partial}{\partial r}(RT) - \frac{\varepsilon_0}{4}\frac{\partial}{\partial r}\frac{|\hat{E}|^2}{\varrho_c}, \quad (8)$$

$$\frac{dT}{dt} = -\frac{2}{3}T\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v) + \frac{2 m_i}{3 k_B}\frac{1}{r^2 R} \times \frac{\partial}{\partial r} r^2\left(\frac{\varphi}{\varrho_c} + \frac{q}{\varrho_c}\right), \quad (9)$$

$$y'' + k^2(1 - R)\,y = 0, \quad (10)$$

where y is given by [13] $y = r\hat{E}$. The equations have to be solved numerically. Doing this for wavelengths of the order of 1 μm involves the considerable difficulty that λ is very different from the hydrodynamic scale lengths. However, we solved the problem by introducing a larger, fictitious wavelength. As can be seen from Eqs. (7)–(10), for equal φ/ϱ_c and q/ϱ_c (i.e. $\varkappa_0/\varrho_c$) (nearly) the same results are obtained as long as λ is taken such that the light-pressure-induced structures have a much smaller scale length than the hydrodynamics and can therefore easily be identified.

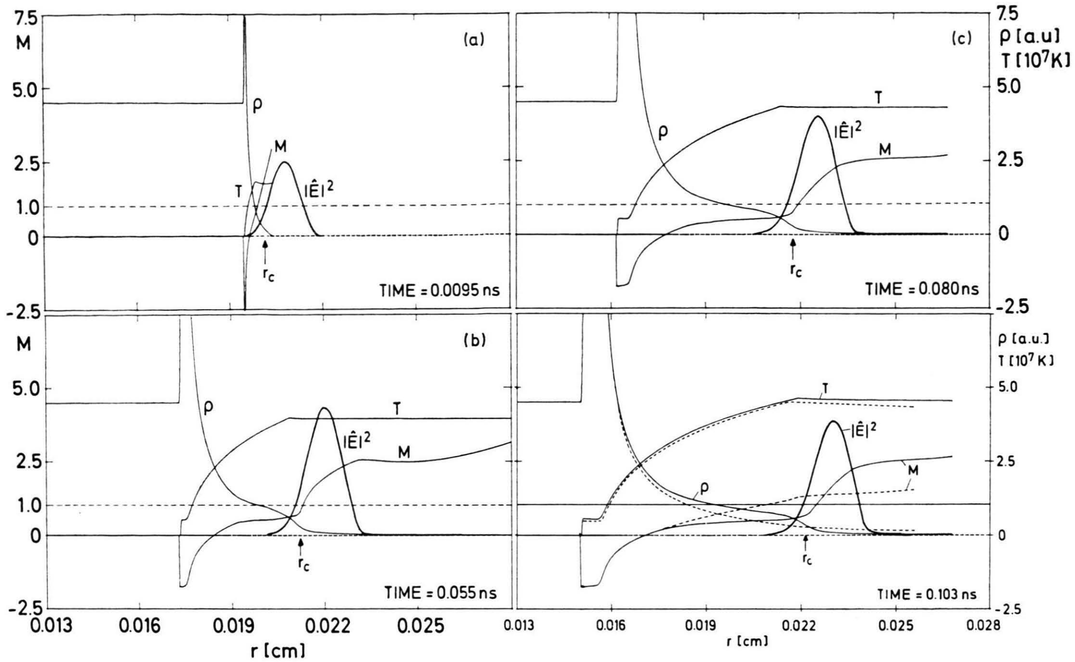

Fig. 1. Formation of an overdense subsonic plateau in a spherical rarefaction wave from a pellet under the action of radiation pressure. ϱ density, T temperature, M Mach number, $\check{E}$ electric field amplitude. Flux limiting factor $f \leqq 0.15$. Dashed curves in the last picture without radiation pressure under otherwise identical conditions.

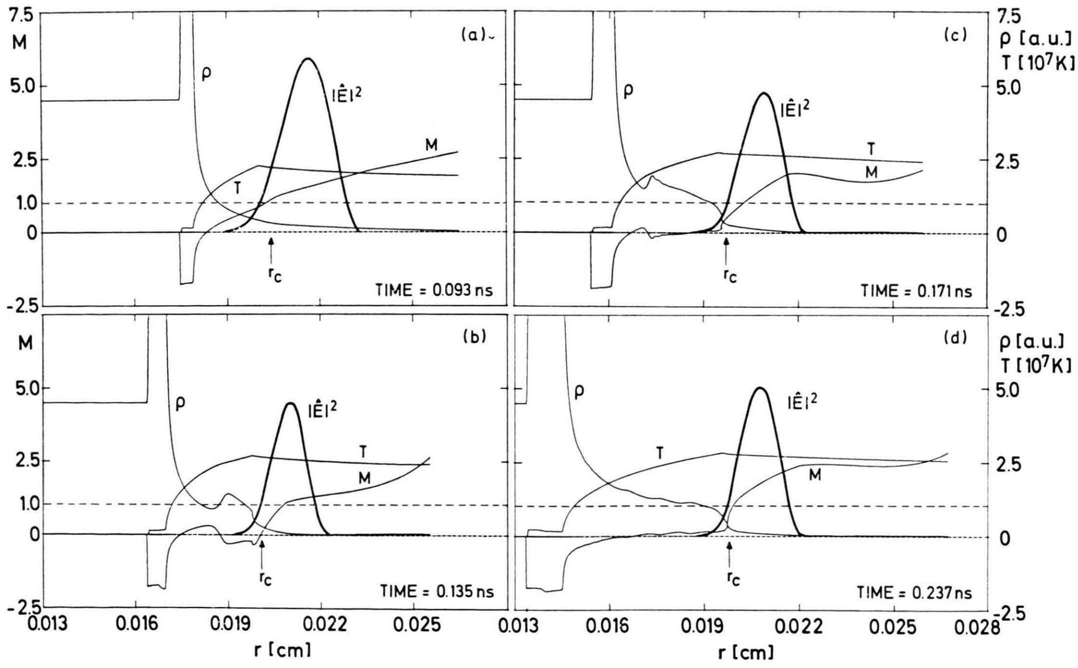

Fig. 2. Plateau formation under the action of light pressure in a rarefaction wave which has already developed before radiation pressure begins to act at the time $t = 0.093$ ns. $f \leqq 0.15$.

In Fig. 1 the numerical result of the radial plasma flow from a pellet under the action of light pressure from the outset is reported. The following values were chosen: pellet radius $r_0 = 200$ µm, $\varrho_0 = 15\,\varrho_c$, $\lambda = 40$ µm, intensity $= 1.2 \times 10^{13}$ W/cm^2, heat flux limit $f \leqq 0.15$. (This corresponds to 1.7×10^{16} W/cm^2 for Nd and 1.7×10^{14} W/cm^2 for CO_2.) The time dependence of φ was assumed to be $\varphi = \varphi_0(1 - e^{-t/\tau})$, $\tau = 10^{-11}$ s. This figure clearly shows the influence of light pressure: it acts in such a way that an overdense plasma plateau forms and the flow remains subsonic in this region. At the critical point a step builds up and transition to supersonic flow occurs in a fraction of a wavelength. When the radiation pressure is disregarded under otherwise identical conditions, the plasma flow clearly becomes supersonic with our parameters in the overdense region (see dashed curves for 0.103 ns in Figure 1). Although higher Mach numbers appear on the right of the critical point when radiation pressure is included, the asymptotic kinetic energy of the plasma is the same in the two cases [14]. The momentum due to ablation and light pressure is balanced by the strong shock on the left running into the undisturbed material. The length of the overdense plateau depends on thermal conduction; with no heat or other energy propagation to the left of r_c the plateau length reduces to zero and the step remains attached to the rarefaction wave at the RHS of the shock. Experimental evidence of the existence of plateau like structures is given in Refs. [15] and [16].

The plateaus represent quasi-steady-state structures, and so formulae (6) for plane flow apply since the width of the density transition $\varrho_1 \to \varrho_2$ is much smaller than r_c. In fact, after transforming the Mach numbers into the frame moving with the critical point, good agreement is found between the numerical results and the analytical expressions. In the moving frame M_1 of Fig. 1 is very small, being typically $M_1 \leqq 0.1$ with light pressure, whereas without the latter the flow proves to be slightly supersonic in this frame.

Numerous computer runs with a variety of different conditions, but with long, smooth laser pulses resulted always in the formation of density plateaus. Density humps were only observed as transient phenomena, as illustrated in Figure 2. To obtain these results the computations of Fig. 1 were repeated without light pressure in the early 95 ps: the first picture of Fig. 2 shows a smooth density curve with $M_c > 1$. Then the light pressure is switched on, a density hump forms and runs into the highdensity region. After 237 ps (last picture) the hump has already reached the leading shock at the extreme left and the overdense subsonic plateaus is completely established.

In the figures reported the wave equation was solved over one standing $|\hat{E}|^2$-maximum only because we know that in the supersonic region the standing wave leads to a density modulation which can be evaluated analytically [9].

[1] H. H. Chen and C. S. Liu, Phys. Rev. Letters **37**, 693 (1976); Phys. Rev. Letters **39**, 1147 (1977); Phys. Fluids **21**, 377 (1978).
[2] P. K. Shukla and K. H. Spatschek, J. Plasma Phys. **19**, 387 (1978).
[3] H. Schamel and K. Elsässer, Plasma Phys. **20**, 837 (1978).
[4] F. J. Mayer, R. K. Osborn, D. W. Daniels, and J. F. McGrath, Phys. Rev. Letters **40**, 30 (1978).
[5] K. Sauer, 7th Int. Conf. Plasma Phys. Contr. Nucl. Fus. Res., Innsbruck, August 1978, paper IAEA-CN-V-2-2.
[6] R. E. Kidder, in Proc. Japan — U.S. Seminar on Laser Interaction with Matter, Yamanaka ed. (Tokyo Int. Book Co., Tokyo 1975), p. 331.
[7] K. A. Brueckner and R. S. Janda, Nucl. Fusion **17**-3, 451 (1977).
[8] K. Lee, D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids **20**, 51 (1977).
[9] P. Mulser and C. van Kessel, Phys. Rev. Letters **38**, 902 (1977).
[10] C. Max and C. McKee, Phys. Rev. Letters **39**, 1336 (1977).
[11] J. Virmont, R. Pellat, and P. Mora, Phys. Fluids **21**, 567 (1978).
[12] Wee Woo and J. S. De Groot, Phys. Fluids **21**, 2072 (1978).
[13] P. Mulser and C. van Kessel, J. Phys. D: Appl. Phys. **11**, 1085 (1978).
[14] P. Mulser and C. van Kessel, Phys. Letters **59 A**, 33 (1976).
[15] R. Fedosejevs, I. V. Tomov, N. H. Burnett, G. D. Enright, and M. C. Richardson, Phys. Rev. Letters **39**, 932 (1977).
[16] H. Azechi, S. Oda, K. Tanaka, T. Norimatsu, T. Sasaki, T. Yamanaka, and C. Yamanaka, Phys. Rev. Letters **39**, 1144 (1977).